Derivatives

1 Differentiate each function.

a) $y = -3x^2 + 4x - 5$

b) $f(x) = 6x^{-1} - 5x^{-2}$

c) $f(x) = 4\sqrt{x}$

d) $y = (3x^2 - 4x)(\sqrt{x} - 1)$

e) $y = \dfrac{x^2 + 4}{3x}$

2 Show that each statement is false.

a) $(f(x)\cdot g(x))' = f'(x)\cdot g'(x)$

b) $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)}{g'(x)}$

3 Determine the equation of the tangent to each function at the given value of x.

a) $f(x) = 2x^2 - 1$ at $x = -2$

b) $g(x) = \sqrt{x} + 5$ at $x = 4$

4. The position of a particle, s metres from a starting point, after t seconds, is given by the function $s(t) = 2t^3 - 7t^2 + 4t$.

a) Determine its velocity at time t.

b) Determine the velocity after 5 s.

5 Determine the points at which the slope of the tangent to $f(x) = x^3 - 2x^2 - 4x + 4$ is zero.

6 A fireworks shell is shot upward with an initial velocity of 28 m/s from a height of 2.5 m.

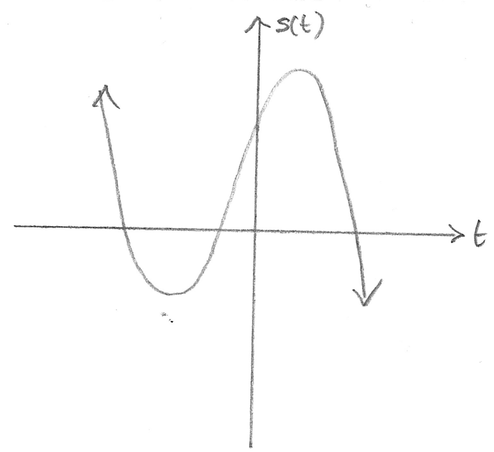
a) State an equation to represent the height of the shell at time t, in seconds.

b) Determine equations for the velocity and acceleration of the shell.

c) State the height, velocity, and acceleration after 2 s.

d) After how many seconds will the shell have the same speed, but be falling downward?

7 Copy this graph of the position function of an object. Draw graphs to represent the velocity and acceleration.



8 Determine the derivative of each function.

a) $y = 2(3x - x^{-1})^2$

b) $g(x) = \sqrt{2x + 5}$

c) $y = \dfrac{1}{\sqrt{3x - 1}}$

d) $\dfrac{-1}{\sqrt[3]{x^2 + 3x}}$

9 Determine an equation of the tangent to $f(x) = x^2(x^3 - 3x)^3$ at the point $(-1, 8)$.

The cost, in dollars, of water consumed by a factory is given by the function $C(w) = 15 + 0.1\sqrt{w}$, where w is the water consumption, in litres. Determine the cost and the rate of change of the cost when the consumption is 2000 L.

10 A car has constant deceleration of 10 km/h/s until it stops. If the car's initial velocity is 120 km/h, determine its stopping distance.

Curve Sketching

Evaluate each limit.

1. a) $\lim\limits_{x\to\infty} (2x^3 - 5x^2 + 9x - 8)$

b) $\lim\limits_{x\to\infty} \dfrac{x + 1}{x - 1}$

c) $\lim\limits_{x\to-\infty} \dfrac{x^2 - 3x + 1}{x^2 + 4x + 8}$

2. Sketch the graph of $f(x)$ based on the information in the table.

x	$(-\infty, -2)$	-2	$(-2, -1)$	-1	$(-1, 0)$	0	$(0, \infty)$
$f(x)$		-5		-1		3	
$f'(x)$	$-$	0	$+$	$+$	$+$	0	$-$
$f''(x)$	$+$	$+$	$+$	0	$-$	$-$	$-$

3 For each function, determine the coordinates of the local extrema, the points of inflection, the intervals of increase and decrease, and the concavity.

a) $f(x) = x^3 + 2x^2 - 4x + 1$

b) $f(x) = \dfrac{3}{4}x^4 - x^3 - x^2 + 5x - 3$

c) $f(x) = \dfrac{3}{x^2 + 1}$

The power, in amps, transmitted by a belt drive from a motor is given by the function $P = 100v - \frac{3}{16}v^3$ where v is the linear velocity of the belt, in metres per second.

a) For what value of v is the power at a maximum value?

b) What is the maximum power?

A ship is sailing due north at 12 km/h while another ship is observed 15 km ahead, travelling due east at 9 km/h. What is the closest distance of approach of the two ships?

The Perfect Pizza Parlor estimates the average daily cost per pizza, in dollars, to be $C(x) = \frac{0.00025x^2 + 8x + 10}{x}$, where x is the number of pizzas made in a day.

a) Determine the marginal cost at a production level of 50 pizzas a day.

b) Determine the production level that would minimize the average daily cost per pizza.

c) What is the minimum average daily cost?

Derivatives of Sinusoidal Functions

1. Differentiate each function.

a) $y = \cos^3 x$

b) $y = \sin(x^3)$

c) $f(x) = \cos(5x - 3)$

d) $f(x) = \sin^2 x \cdot \cos\left(\frac{x}{2}\right)$

e) $f(x) = \cos^2(4x^2)$

f) $g(x) = \frac{\cos x}{\cos x - \sin x}$

2. Find the local maxima, local minima, and inflection points of the function $y = \sin^2 x - \frac{x}{2}$.

3. The height above the ground of a rider on a large Ferris wheel can be modelled by $h(t) = 10\sin\left(\frac{2\pi}{30}t\right) + 12$, where h is the height above the ground, in metres, and t is time, in seconds. What is the maximum height reached by the rider, and when does this first occur?

4. A weight is oscillating up and down on a spring. Its displacement, from rest is given by the function $d(t) = \sin 6t - 4\cos 6t$, where d is in centimetres and t is time, in seconds.

a) What is the rate of change of the displacement after 1 s?

b) Determine the maximum and minimum displacements and when they first occur.

Exponential and Logarithmic Functions

a) Compare the graphs of $y = e^x$ and $y = 2^x$.

b) Compare the graphs of the rates of change of $y = e^x$ and $y = 2^x$.

c) Compare the graphs of $y = \ln x$ and $y = e^x$, and their rates of change.

Evaluate, accurate to two decimal places.

a) $\ln 5$ **b)** $\ln e^2$ **c)** $(\ln e)^2$

Simplify.

a) $\ln(e^x)$ **b)** $e^{\ln x}$ **c)** $D^{\ln D^x}$

Determine the derivative of each function.

a) $y = -2e^x$ **b)** $g(x) = 5 \cdot 10^x$

c) $h(x) = \cos(e^x)$ **d)** $f(x) = xe^{-x}$

Determine the equation of the line perpendicular to $f(x) = \frac{1}{2}e^{x+1}$ at its y-intercept.

Radium decays at a rate that is proportional to its mass, and has a half-life of 1590 years. If 20 g of radium is present initially, how long will it take for 90% of this mass to decay?

Determine all critical points of $f(x) = x^2e^x$

The power supply, in watts, of a satellite is given by the function $P(t) = 200e^{-0.001t}$, where t is the time, in days, after launch. Determine the rate of change of power

a) after t days **b)** after 200 days

The St. Louis Gateway Arch is in the shape of a catenary defined by the function

$$y = -20.96\left(\frac{e^{0.0329x} + e^{-0.0329x}}{2} - 10.06\right),$$ with

all measurements in metres.

a) Determine an equation for the slope of the arch at any point x m from its centre.

b) What is the slope of the arch at a point 2 m horizontally from the centre?

c) Determine the width and the maximum height of the arch.