Calculus Study Plan

1. Limits

Review the methods used to evaluate limits and the situations in which they are used.

- substitution
- graphically
- algebraic techniques such as factoring and expansion
- rationalization
- change of variable
- ☺one sided limits
- ☺limits to infinity
- ☺L'Hopitals Rule (limits to infinity and one sided limits are needed for curve sketching others for higher functions)

2. Power Sum and Difference rules for derivatives — Text Pg 145

Find the derivative if y equals

1. $3x+7$
2. $3x^7-3x+10$
3. $\frac{1}{4}x^4-\frac{1}{3}x^3+\frac{1}{2}x^2+10$
4. $4x^{-4}+5x^{-5}+6x^{-6}$
5. $(3x+2)(2x-3)$
6. $\left(\frac{2}{3}x^2\right)^2+\left(4x^3\right)^3$
7. $1-\frac{1}{x}+\frac{2}{x^2}$
8. $\frac{3x^3+4x^4-5x^5}{x^2}$
9. $\frac{5}{x}+\sqrt[5]{x}+\sqrt{5x}$

10. Find the area of the triangle formed by the three points where the slopes of the tangent lines to the curve $y=x^4-2x^2+4$ are zero at those points.

3. Product and Quotient Rule — Text Pg 150

Find the derivative if y equals

A

1. $(x^2+1)(x^2+2)$
2. $5x(x^2+3)$
3. $\sqrt[3]{x^2}\left(x^4-5(\sqrt[3]{x^5})\right)$
4. $(x^2-3x+4)(2x^2+4x)$
5. $(2+x^3)\left(5-\frac{1}{x^2}\right)$
6. $(2x^2+3x^3)^2$
7. $(x^2+1)(x^2+2)(x^2+3)$
8. $(4x+1)(x+2)$ at $x=-2$
9. $(3x^2+6x+2)(4-x)$ at $x=-1$

B

1. $\frac{1}{x}$
2. $\frac{4x-1}{x+9}$
3. $\frac{1}{x^2-x+1}$
4. $\frac{x^2+3x+3}{x+1}$
5. $\frac{x+3}{4-x^2}$
6. $\frac{3x^2+9x+4}{x^2+3x+1}$
7. $\frac{3x^3+2x^2-x+6}{x^3}$
8. $\frac{7\sqrt{x}-10}{1-\sqrt{x}}$
9. $\frac{x^2+3x}{x^2-5x+1}$
10. $\frac{(2x+5)(3x+5)}{x+1}$

C

Find the equation of the tangent to the curve $y=\frac{4x+8}{x+1}$ *which is parallel to* $4x+y=7$.

Given $f(x)=\frac{ax}{x^2+b}$ Find a an b such that $f'(1)=0$ *and* $f'(0)=1$

Find the equation of the tangent and normal to the curve $y=\frac{x^2+2}{x^2-2}$ at the point (1, -3)

If $f(1)=2,\ f'(1)=3,\ g(1)=4,\ g'(1)=5,$ evaluate $\frac{d\left(\frac{f(x)}{g(x)}\right)}{dx}$ at $x=1$.

4. The Chain Rule

Text pg 158-9

Find the derivative

1. $y=(1+x)^{10}$
2. $y=\left(7x^3+6x\right)^5$
3. $y=\sqrt{x^2-1}$
4. $y=\left(x^3+2\right)^{-2}$
5. $y=\left(2x^3+1\right)^4\left(4x^2+7\right)^2$
6. $y=\sqrt{5+\sqrt{x}}$
7. $y=\left(\frac{x+7}{x+5}\right)^5$
8. $y=\frac{7x}{\sqrt{5-2x}}$
9. $y=(x+1)\sqrt{x^2+2x+2}$
10. $y=\sqrt{x+\sqrt{x+\sqrt{x}}}$
11. $y=[x^5+(3x^2-2)^4]^6$
12. $\sqrt[3]{(1+4x)^4+x^5}$

13. If $h(x)=f(g(x))$, *and* $f(3)=f'(3)=g'(3)=g(3)=3$, *find* $h'(3)$.
14. $y=\frac{u+3}{u-1}$, and $u=x^{\frac{1}{3}}$, find $\frac{dy}{dx}$ at $x=8$.
15. Find the equation of the tangent line to the curve $y=(x^3-1)^{10}$ at the point $(1, 0)$
16. Find the equation of the tangent line to the curve $y=(4+3x^2)^{\frac{1}{4}}$ at $x=2$.
17. Find the tangent and normal to the curve $y=\sqrt[3]{x+64}$ at the point where the curve cut the y-axis.
18. *Given* $x^3f(x)+\left(f(x)\right)^3+f\left(x^3\right)=3$ *and* $f(1)=2$, *find* $f'(1)$.
19. Find the area of the triangle formed by the three points on the curve $y=\left(\frac{x^2-1}{x^2+1}\right)^2$ where the slopes of the tangent lines to the curve at those points are zero.
20. Find the equations of the tangent and normal to the curve $y=\sqrt[3]{\left(x^3-2x+4\right)^2}$ *at* $x=2$.
21. Given $f\left(x^3-1\right)=4x^2$. *Find* $f'(0)$.
22. Find $f'(x)$ in terms of g *and* g' in the following cases
    a) $f(x)=g\left(x^n\right)$ *b)* $f(x)=\left(g(x)\right)^n$ *c)* $f(x)=x^ng\left(x^n\right)$ *d)* $f(x)=g\left(g(x)\right)$

5. Implicit Differentiation

Text pg. 178 1-11

Find $\frac{dy}{dx}$

1. $y^2+x=0$
2. $x^2+4y^2=36$
3. $x^3y^4=3$
4. $xy+y^3=x^2+x+1$
5. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$
6. $\sqrt{x}-\sqrt{y}=\sqrt{a}$
7. $y\sqrt{x}+x\sqrt{y}=10$
8. $y^2=\frac{x+y}{x-y}$
9. $\frac{x^3}{y^3}+\frac{y^3}{x^3}=1$
10. Find the equation of tangent to $y^2+2y+10x+11=0$ which to parallel to 2y+x = 0.
11. Find the tangent and normal to the curve $x^3-6xy-2y^3=0$ at (4, 2).
12. Find the tangent and normal to the curve $x^2=y^2x+2$ *at* $x=-1$.
13. Find the points on $\frac{x^2}{16}-\frac{y^2}{9}=-1$ whose tangents are either vertical or horizontal.
14. Find the equation of the tangent to $\sqrt{x}+\sqrt{y}=2\sqrt{a}$ which is parallel to $x+y=2$

6. Velocity and Acceleration

Text pg 186 4-14

A For the position function $s=3t^3-7t^2-2t,\ t\geq 0$, choose the correct answer for t = 2.

a) Is s increasing or decreasing?
b) Is v increasing or decreasing?
c) Is the particle moving in the positive or negative direction?
d) Is the object moving away or towards the origin?
e) Is the object speeding up or slowing down?

B A man drives a car at a speed of 48 m/s sees a stop sign 130 m away. He presses the brake and his position function is $s=48t-t^3$. Will he go beyond the stop sign?

7. Related Rates

Text pg 193-195

Remember the Algorithm

Given Rate(s)
Required rate(s)
Relationship
Relationship between the Rates
Substitution

1. The side of a cube increases at 1 cm/s. How fast is the diagonal of the cube changing when the side is 1 cm?

2. Radius of a circle varies as time t in m/s according to the following rule $r = t^3 + 2t$. Find the rate of change of the area at $t = 2$.

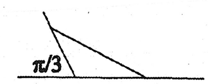
3. Water is added to a cylindrical tank of radius 5 m and height 10 m at a rate of 100 L/min. Find the rate of change of the water level when the water is 6 m deep. ($1\ L = 1000\ cm^3$)

4. A spherical balloon is inflated at a rate of $4\pi\ cm^3/s$. At the moment the radius is 5 cm, find the rate of change of
   a) the radius b) the surface area

5. A balloon is rising at a rate of 200m/min. For every 500 m increase in height the air pressure causes the radius to increase 2 cm. Find the rate of increase of the volume when the radius is 10 m.

6. The dimensions of a conical tank is of radius 3 m and height 6 m. Water is added to it at a rate of $\pi\ m^3/min$. Find the rate of change of the water level when the height is 3 m.

7. A wall is inclined 60 degree to the ground. A ladder $4\sqrt{3}$ m is resting on the wall. The foot of the ladder is moving away at a rate of 2 m/s. How fast is the top sliding down at the instant when the foot of the ladder is 4 m away from the foot of the wall?



8. At noon ship A leaves a location and is moving due east at the rate of 10 km/h. One hour later another ship B starts to leave the same location and is moving at a rate of 15 km/h in the direction N30°E. Find the rate of change of distance between the two ships at 3:00 p.m.

9. A trough is 10 m long and its ends are isosceles trapezoids with base 2 m, top 3 m, and height 4 m. Water is added at a rate of $5\ m^3/min$. Find the rate of change of the water level when the water is 2 m deep.

10. A spot light on the ground is shining on a vertical building which is 20 m from the spot light. A man 2 m tall is walking away from the light and is walking directly towards the building at a rate of 4 m/s. Find the rate of change of the length of his shadow on the building when he is 5 m from the light.

8. Optimization Problems

Text pg 206-7

Remember the Algorithm

Quantity to be Optimized
Relationship
Reduce to 2 variables if necessary
Differentiate w.r.t. the variable on the rhs.
Find locals
Check

1. Sum of two positive numbers is 12. Find the two numbers such that the product of one and the cube of the other is maximum.

2. A rectangle is inside a parabola $y = 12 - x^2$ with its base lying on the x-axis and the other two vertices lie on the parabola. Find the dimension of the largest rectangle.

3. A rectangular field is to have 100 m^2 in area. It was enclosed by fence. The north-south sides costs $20/m, east-west sides cost $5/ m. What are the dimensions so that the cost is minimum?

4. A rectangular poster is of area 6912 cm^2. Side margin is 8 cm each and top and bottom margin are each 6 cm. Find the dimensions of the poster that give the maximum printing area.

5. Squares of equal length are cut from the 4 corners of a square sheet of side 12 cm. The four sides are then folded to form an open top box. Find the length of the side of the squares cut that give a maximum volume.

6. The total surface area of a squared base open top rectangular box is 12 square units. Find the dimensions of the box such that the volume is maximum.

7. A window consists of a rectangle surmounted by a semicircle. The perimeter is 8 m. Suppose rectangular part admit twice as much sunlight as the circular part. Find the width of the window which admit most sunlight.

8. The cost of laying power line underwater is 3 times that of underground. An island is 4 km from the shore and a power station is at distance 8 km from the point on the shore which is closest to the island. How should the power line be laid so that the cost is minimum

9. A ship A leaves a dock at noon and travels due south at 20 km/h. Another ship B has been heading due east at 30 km/h and reach the same dock at 3:00 p.m . At what time will the two ship be closest?

9. Curve Sketching

Text pg 369-70, 375

Remember the Algorithm
holes
Intercepts
Domain
Symmetry
Critical Numbers
Points of Inflection
Interval Chart
VA(s) and in vicinity
HA and in vicinity
SA (if necessary) or End Behaviour.
Range
Sketch

1. $y=\frac{x}{(x+1)^2}$ 2. $y=e^{-x^2}$ 3. $y=\frac{\ln x}{x}$ 4. $y=x-\ln x$ 5. $y=xe^{-x^2}$

10. Exponential and Logarithmic Functions

Text pg 302-3, 309-10 315-16

A Find the derivatives of the following functions, given y equals .

1. e
2. e^{x+7}
3. e^{4x}
4. e^{-x}
5. e^{4x+7}
6. $e^{-\frac{1}{3}x+8}$
7. $e^{\sqrt{x+3}}$
8. $\sqrt{e^{x+3}}$
9. $x^{\sqrt{e}}+\sqrt{x^e}+e^{\sqrt{x}}+\sqrt{e^x}$
10. e^{x+e^x}
11. $\sqrt{1+e^{kx}}$
12. $\frac{1}{e^x}+e^{\frac{1}{x}}$
13. e^{e^x}
14. $\frac{e^x-e^{-x}}{e^x+e^{-x}}$
15. $7^{\sqrt{x}}$
16 3^{kx}
17. $9^{\sqrt{x^3+8}}$
18. $7^x e^x$

B Find the derivatives of y

1. $e^y+x^3=3x$
2. $e^{xy}+y=1$
3. $ye^y=4x$
4. $\ln(x+y)=e^{x-y}$
5. $e^{y^2}=xy$
6. $\ln(xy)=e^x$
7. $x\ln y+x=y^2$
8. $y=\ln(x^2+y^2)$
9. $e^y\ln y=x^2$
10. $e^{xy}-x^3+3y^2=1$
11. $x\ln y=x^2$
12. $y^3+xe^y=3x^2-4$

C

1. Find the equation of the tangent to the curve $y = xe^x + e^x + 1$ at $(-1,1)$

2. The position of an object is given by the equation $s = (t^2 - 3)e^{1-t}$, $t \geq 0$. s in metres and t in seconds
   a) When is the velocity zero? When is it positive?
   b) When is the acceleration zero? When is it positive?

3. ABCD is a rectangle with AB lies on the x-axis and vertices C, D on the curve $y = e^{-x^2}$. Find the maximum area of rectangle ABCD.

D

Find the derivative of the following functions, given y

1. $\ln 7$
2. $\ln 7x$
3. $\ln(7x+8)$
4. $\ln(7x+8)^3$
5. $\ln(7x+8)^{\frac{1}{7}}$
6. $\ln\sqrt[3]{7x^2+8}$
7. $\ln\left(\frac{1}{x}\right)+\frac{1}{\ln x}$
8. $\sqrt{\ln x} - \ln\sqrt{x}$
9. $\ln\sqrt{\frac{1-x}{1+x}}$
10. $\ln\sqrt{\frac{x^2+1}{x^2-1}}$
11. $e^{\ln x}$
12. $e^{\ln(x^3+7x^2+6)}$
13. $\sqrt{\ln(x^2+1)}$
14. $\ln(\ln x + x)$
15. $\ln\left(\frac{\sqrt{x^2+1}}{(9x-4)^2}\right)$
16. $\ln\left(x+\sqrt{1+x^2}\right)$
17. $\ln\left(\frac{x^4}{(3x-4)^2}\right)$
18. $(x^8 \ln x)^9$
19. $x^k \ln x$
20. $\log_{10}(9x^2+7)$
21. $\log_3(5x^2+4)$
22. $\left(\ln(x^k+1)\right)^m$
23. $\frac{\ln x}{x}$
24. $(\ln(\ln x))^4$
25. $\ln\left(\frac{1-e^x}{1+e^x}\right)$
26. $a^{\frac{x}{\ln x}}$
27. $\sqrt{\ln(e^{2x}-e^{-2x})}$

E

Find the derivatives of y, given y equals

1. x^x
3. $e^x + \pi^e + \pi^x + x^\pi + x^x$
3. $x^{\frac{1}{x}}$
4. $(\ln x)^x$
5. $(x^x)^x$
6. $x^{\ln x}$
7. $y = \frac{1}{x+1}\sqrt{\frac{(x-1)(x+3)}{x+1}}$
8. $y = \sqrt{(3x^2+8)\sqrt{4x+1}}$
9. $y = \frac{e^x\sqrt{x^2+3}}{(x^3+4)^5(x^2+1)^2}$

11. Trigonometric Functions

Find the derivatives, given y

Text pg 400, 403-4

1. $\cos(3x+7)$
2. $\tan(7x+9)$
3. $x^2 - \cot x$
4. $\frac{\cos x}{x^2}$
5. $\sin(x^2+2)$
6. $\cot(x^4+2x)$
7. $\frac{2-\sin x}{2+\sin x}$
8. $\sec^3(\sqrt{x})$
9. $x^3\cos\frac{1}{x}$
10. $\csc^2 2x$
11. $\sqrt{\cot 2x}$
12. $\frac{\tan x}{x}$
13. $\sqrt{\sin x}$
14. $\frac{x^3}{\tan 3x}$
15. $x\sin(1-x^2)$
16. $\sqrt{\sin^2 x}$ (*Note:* $\neq \sin x$)
17. $x^2 - \cot x$
18. $x^3\csc 6x$
19. $\sqrt{\csc 2x}$
20. $\sin^3 3x + \cos^2 2x$
21. $\csc(x^2+4)$

12. Antiderivatives

See sheets from class