4.7 Counting Techniques and Probability Strategies—Combinations

1. Evaluate each of the following.

   (a) C(8, 3) (b) ${}_2C_1$ (c) $\binom{10}{4}$ (d) C(21, 13) (e) ${}_5C_2$

2. Solve for n.
   (a) $C(n, 2) = 66$ (b) $6P(n + 1, 3) = 48C(n, 4)$

3. In how many ways can a child select three different ice cream flavours from nine different ice cream flavours?

4. How many hands of six cards can be selected from a standard deck of 52 cards?

5. In how many ways can a group of five people be chosen from seven couples (each of which has one male and one female) to form a club, given each of the following conditions?
   (a) All are equally eligible for the club.
   (b) The club must include two females and three males.

6. If a closed polygon has 21 diagonals, how many sides does it have?

optional 7. Prove that $C(n + 1, r) = C(n, r) + C(n, r - 1)$.

8. Monique has eight red jelly beans and six purple jelly beans in a jar. She pulls out one jelly bean.
   (a) What are the odds in favour of the jelly bean being a red one?
   (b) What are the odds against the jelly bean being a purple one?

9. The weather forecaster predicts that the probability of sun tomorrow is 60%. What are the odds in favour of sun tomorrow? Explain your answer.

10. Two teachers and six students on a class trip must ride in two four-passenger cars.
    (a) What is the number of ways that the eight people can be divided into two groups to ride in the two cars?
    (b) What is the number of ways if only teachers are allowed to drive?
    (c) What is the probability that the teachers will ride in the same car?

Answers

1. (a) 56 (b) 2 (c) 210 (d) 203 490 (e) 10 2. (a) $n = 12$ (b) $n = \frac{-2 \pm \sqrt{41}}{4}$; no solution

3. $C(9, 3) = 84$ 4. $C(52, 6) = 20\,358\,520$ 5. (a) $C(14, 5) = 2002$ (b) $C(7, 2) \times C(7, 3) = 735$
6. The polygon has 7 sides. 7. Start with the right-hand side and find a common denominator of $(n-r+1)(n-r)!r(r-1)!$. In the numerator, factor out an $n!$ and you are left with $(n + 1)$. Thus, $n!(n + 1)$ gives $(n + 1)!$ in the numerator. The denominator simplifies to $(n + 1 - r)!r!$. This quotient equals the left-hand side. Thus proven.

8. (a), (b) 4:3 9. 3:2 10. (a) $C(8, 4) = 70$ (b) $C(2, 1) \times C(6, 3) = 40$ (c) $1 - \frac{40}{70} = \frac{3}{7}$