4. A horizontal gutter is to be made from a long piece of sheet iron 8 in. wide by turning up equal widths along the edges into vertical position. How many inches should be turned up at each side to yield the maximum carrying capacity?
5. The difference between two numbers is 20. Select the numbers so that the product is as small as possible.
6. A box with a square base is to have an open top. The area of the material in the box is to be 100 in^2. What should the dimensions be in order to make the volume as large as possible? What is the result for an area of S square inches?
7. A Norman window is in the shape of a rectangle surmounted by a semicircle. Find the dimensions when the perimeter is 12 ft and the area is as large as possible.
8. Find the radius and central angle (in radians) of the circular sector of maximum area having a perimeter of 16 in.
9. The top and bottom margins of a page are each $1\frac{1}{2}$ in. and the side margins are each 1 in. If the area of the printed material per page is fixed at 30 in^2, what are the dimensions of the page of least area?
10. A closed right circular cylinder (i.e., top and bottom included) has a surface area of 100 in^2. What should the radius and altitude be in order to provide the largest possible volume? Find the result if the surface area is S in^2.
11. A right circular cone has a volume of 120 in^3. What shape should it be in order to have the smallest lateral surface area? Find the result if the volume is V in^3.
12. At midnight, ship B was 90 mi due south of ship A. Ship A sailed east at 15 mi/hr and ship B sailed north at 20 mi/hr. At what time were they closest together?
13. Find the coordinates of the point or points on the curve $y = 2x^2$ which are closest to the point (9, 0).
14. Find the coordinates of the point or points on the curve $x^2 - y^2 = 16$ which are nearest to the point (0, 6).
15. (a) A right triangle has hypotenuse of length 13 and one leg of length 5. Find the dimensions of the rectangle of largest area which has one side along the hypotenuse and the ends of the opposite side on the legs of this triangle. (b) What is the result for a hypotenuse of length H with an altitude to it of length h?
16. A trough is to be made from a long strip of sheet metal 12 in. wide by turning up strips 4 in. wide on each side so that they make the same angle with the bottom of the trough (trapezoidal cross section). Find the width across the top such that the trough will have maximum carrying capacity.

17. The sum of three positive numbers is 30. The first plus twice the second plus three times the third add up to 60. Select the numbers so that the product of all three is as large as possible.

18. The stiffness of a given length of beam is proportional to the product of the width and the cube of the depth. Find the shape of the stiffest beam which can be cut from a cylindrical log (of the given length) with cross-sectional diameter of 4 ft.

19. (a) A manufacturer makes aluminum cups of a given volume (16 in^3) in the form of right circular cylinders open at the top. Find the dimensions which use the least material. (b) What is the result for a given volume V?

20. In Problem 19, suppose that the material for the bottom is $1\frac{1}{2}$ times as expensive as the material for the sides. Find the dimensions which give the lowest cost.

21. Find the shortest segment with ends on the positive x and y axes which passes through the point $(1, 8)$.

22. The product of two numbers is 16. Determine them so that the square of one plus the cube of the other is as small as possible.

23. Find the dimensions of the cylinder of greatest lateral area which can be inscribed in a sphere of given radius R.

24. A piece of wire of length L is cut into two parts, one of which is bent into the shape of a square and the other into the shape of a circle. (a) How should the wire be cut so that the sum of the enclosed areas is a minimum? (b) How should it be cut to get the maximum enclosed areas?

25. Find the dimensions of the right circular cone of maximum volume which can be inscribed in a sphere of given radius R.

26. A silo is to be built in the form of a right circular cylinder surmounted by a hemisphere. If the cost of the material per square foot is the same for floor, walls, and top, find the most economical proportions for a given capacity V.

27. Work Problem 26, given that the floor costs twice as much per square foot as the sides and the hemispherical top costs three times as much per square foot as the sides.

28. A tank is to have a given volume V and is to be made in the form of a right circular cylinder with hemispheres attached at each end. The material for the ends costs twice as much per square foot as that for the sides. Find the most economical proportions.

29. Find the length of the longest rod which can be carried horizontally around a corner from a corridor 8 ft wide into one 4 ft wide. [*Hint:* Observe that this length is the minimum value of certain lengths.]

13. No maximum or minimum. [For interval $-3 \leq x \leq 3$, max. is (3, 11), min. is (−3, −49).]
15. No maximum or minimum
17. Max. at $(2, \frac{1}{3})$; no minimum

Section 6

1. Length = width = 30 feet; length = width = $\frac{1}{4}L$
3. $h = \frac{2}{3}\sqrt{3}\,R, r = \frac{1}{3}\sqrt{6}\,R$
5. 10, −10
7. Radius of semicircle = height of rectangle = $12/(4 + \pi)$ ft.
9. Width = $(2 + 2\sqrt{5})$ in.; height = $(3 + 3\sqrt{5})$ in.
11. $r = \sqrt[6]{360^2/2\pi^2}, h = \sqrt[3]{360\cdot 2/\pi}$; $r = \sqrt[6]{9V^2/2\pi^2}, h = \sqrt[3]{6V/\pi}$
13. (1, 2)
15. a) Base = $\frac{13}{2}$; height = $\frac{30}{13}$
    b) Base = $\dfrac{H}{2}$; height = $\dfrac{h}{2}$
17. Each number equals 10
19. (a) $r = h = \sqrt[3]{16/\pi} = 2\sqrt[3]{2/\pi}$ in.; (b) $r = h = \sqrt[3]{V/\pi}$ in.
21. From (5, 0) to (0, 10)
23. Radius = $\frac{1}{2}R\sqrt{2}$; height = $R\sqrt{2}$
25. Height = $\frac{4}{3}R$; radius of base = $\frac{2}{3}\sqrt{2}R$
27. $r = (3V/20\pi)^{1/3}$; $h = 6(3\,V/20\pi)^{1/3}$
29. $4(1 + 2^{2/3})^{3/2}$ feet.

Section 7

1. $df(x, h) = (4x^3 + 6x - 2)h$
3. $df(x, h) = \dfrac{(4x^5 + 2x)h}{\sqrt{x^4 + 1}}$
5. $df(x, h) = \dfrac{(8x^2 + 3x + 4)h}{3(x^2 + 2)^{1/2}(2x + 1)^{2/3}}$
7. $df = 0.03$; $\Delta f = 0.0301$
9. $df = 0.18$; $\Delta f = 0.180901$
11. $df = -0.01250$; $\Delta f = -0.01220$
13. $df = -0.05$; $\Delta f = -0.04654$
15. 8.0625
17. 1.02000
19. 1.98750
21. 0.049
23. $1\frac{2}{3}\%$ (approximately)
25. $6a^2t$ in^3 of paint

Section 8

1. $15x(x^3 - 3x^2 + 2)^4(x - 2)$
3. $-5(x^3 + 4)^{-6}(3x^2)$
5. $(5x^2 + 6x)(2x + 3)^{1/2}$
7. $2(x + 1)(2x - 1)^2(5x + 2)$
9. $2(1 - x^2)(x^2 + 1)^{-2}$
11. $\frac{2}{3}x^{-1/3}(x + 1)^{-5/3}$
13. $-(x + 1)^{-1/2}(x - 1)^{-3/2}$
15. $(4x + y + 2)/(-x + 2y + 3)$
17. $-y^{1/2}/x^{1/2}$
19. $(6x^2 - y^2 + 2)/(2xy + 3y^2 + 1)$
21. $-8(2u^2 + 2u + 1)r(r^2 + 5)^3/(u^2(u + 1)^2)$