

1. Water is flowing into a vertical cylindrical tank at the rate of 12 cubic meters per minute, if the radius of the tank is 3 meters, how fast is the surface rising?
2. A rectangular prismatic tank has the following dimensions: length of 10m, width of 6m, and depth of 8m. As it is being filled with water, it is noted that the surface level of the water is rising at the uniform rate of $6\text{cm}/\text{min}$. What is the rate of inflow into the tank?
3. The cross-section of a water trough is an equilateral triangle. Water is pumped into this 10m long trough at $10\text{m}^3/\text{s}$. Find the rate at which the depth of the water is increasing when the water is 1m deep.
4. ~~A conical glass vase is being filled with water at a rate of 10cm^3 per second. The vase is 20cm high and 3cm in radius at the top. Find the rate at which the level is rising when the depth is 10cm.~~
5. Water flows out of a conical funnel at $3\text{cm}^3/\text{s}$. If the top of the funnel is 8cm in diameter and the height is 8cm, find the rate of descent of the level of the liquid when it is 2cm from the top.
6. Water pours at the rate of 2 cubic cm per second into a conical cup of height 10cm, having radius of top 3cm. How fast is the water level rising when it is 4cm below the top?
7. Wheat falls from an elevator chute at the rate of 50m^3 per minute and piles itself into a conical pile, which has a height equal to two-thirds of the base. How fast is the cone growing in height when the radius of the base is 10m?
8. Sand is poured from a conveyor belt onto a conical pile at the rate of $36\pi\text{m}^3$ per minute. The radius of the base of the circular conical pile is twice the height, h , at time, t , minutes, while the volume of the conical pile is given by the formula $\frac{1}{3}\pi r^2 h$.
 - (a) find h at time t minutes, assuming the pouring began at $t=0$.
 - (b) How fast is h increasing eight minutes after pouring began?
 - (c) When is h equal to 12m?
 - (d) How fast is h increasing at the instant when h is equal to 12m?
9. A cistern is in the shape of a cone, 12m deep and 6m in diameter at the top. If 10m^3 of water flows into the cistern each minute, and the level of water is increasing $6\text{cm}/\text{min}$. when the water is 8m deep, how much water is leaking away each minute?
10. Grain, flowing from a spout at a constant rate of $1500\text{cm}^3/\text{min}$, forms a conical pile on a level floor, the angle of the vertex of the cone being 120° . Find the rate at which the radius of the base is increasing and the rate at which the floor surface the grain covers is increasing at the instant the radius of the base is 5m.

MCB4U

SUPPLEMENTARY RELATED RATES PROBLEMS

11. Water is leaking out of an inverted circular conical tank at a rate of 2m^3 per minute. The tank is 30m in diameter at the top. It is 20m deep at the deepest point, which is the vertex of the cone and is on the vertical centre axis of the tank. If the water level is rising at the rate of 0.05m per minute when the greatest depth is 10m, find the rate at which water is then being poured into the tank.
12. A spherical balloon is being inflated at a rate of $3\pi\text{m}^3$ per minute. Twelve minutes after the inflation first begins, what is the rate of increase of the diameter of the balloon with respect to time?
13. Gas escapes from a spherical balloon at the constant rate of 1m^3 per minute. How fast is the radius decreasing and how fast is the surface area of the balloon shrinking when the radius is 10m?
14. A weather balloon rises through the air at a rate of 50m per minute. Every thousand meters the decrease of air pressure outside the balloon causes its radius to increase by 10cm. How rapidly is the volume increasing at the instant its radius is 3m?
15. ~~Fruit punch is being ladled out of a hemispherical bowl of radius $\frac{1}{2}\text{m}$ at a rate of $\frac{1}{10}\text{m}^3$ per minute. How fast is the level of the fruit punch falling at the time the depth at the centre is 40cm?~~
16. A ladder 10m long leans against a vertical wall. If the bottom slides out at the rate of 1m per minute, how fast is the top descending when the bottom is 6m away from the wall?
17. A fire ladder 26m long leans against a vertical wall. The bottom is shifted toward the wall at the rate of 5 m per minute. How fast is the midpoint of the ladder rising at the instant it is 5 m from the wall?
18. An extension ladder 20 m long is propped against a vertical wall. The ladder slides down, the foot of the ladder moving at 1m per second. Find the rate of descent of the top of the ladder at the following times:
 - (a) when the bottom of the ladder is 12m from the wall,
 - (b) when the ladder makes an angle of 60° with the ground, $\angle = 60^\circ$
 - (c) when the top of the ladder is 16m above the ground.
19. A 25m ladder is pulled away from a house by pulling the foot of the ladder away at 1m/sec. How fast is the top of the ladder approaching the ground when the bottom of the ladder is 15m from the house?
20. A man 2m tall walks away from a lamppost 5m high at the rate of $4\text{km}/\text{h}$. How fast does the end of his shadow move?

21. A pedestrian 2m tall runs directly away from a streetlight 6m above the ground at 4m per second. Determine the following the instant he is 24m from the base of the light post.
- the speed of the end of his shadow.
 - the rate of increase in length of his shadow.
22. At noon, a vessel is sailing due north at 15 knots. A second vessel, 30 nautical miles due north of the first vessel, is sailing due east at 20 knots. Describe how the distance between the two vessels is changing at the end of 1 hour.
23. Ship A approaches a harbor entrance from the north at 15 knots, while ship B approaches from the west at 18 knots. How rapidly are the ships approaching each other at the instant both are 5 nautical miles from the entrance?
24. Two ships (A and B) steam eastward on parallel tracks at 12 knots and 15 knots respectively. Ship A started at 9 a.m., while ship B started out 3 hours later, and with a starting point 6 nautical miles due south of ship A. Are the ships approaching or separating, and at what rate, at 10 p.m., midnight and 2 a.m.
25. A ship is moving through still water with a speed of $20 \frac{m}{min}$. A man in the stern, 5m above water, is pulling in a rowboat attached to the ship by a rope. If he pulls the rope in at $20 \frac{m}{min}$, how fast is the rowboat moving through the water when there is 52m of rope out?
26. A ship steaming due west at 10 knots crosses the track of a second ship steaming due north at 16 knots a quarter hour after the second ship passed through the same point. How fast is the distance between the two ships increasing one half hour after the westbound ship crosses the track?
27. A train, starting at noon, travels north at 40km per hour. Another train, starting from the same point at 2 p.m. travels east at 50km per hour. Find, the nearest km per hour, how fast the two trains are separating at 3 p.m.
28. A jet flying north at 600 km per hour passes over a town at 12 noon exactly. A second jet flying east at 540km per hour passes over the town 1 minute after 12. If the altitudes of the two aircraft are the same, how fast are they moving apart at 6 minutes after 12.
29. A car traveling at 40m per second crosses a railroad track 4 seconds ahead of a train that is approaching at a speed of 30m per second. If the paths are at right angles, at what rate is the distance between car and train changing two seconds after the car crossed the track?
30. A kite flying at 30m is blown horizontally by the wind at a speed of 4m per second. If the string is played out from a fixed position, how fast is the length of the string increasing when it is 50m long?

31. An observer is 4km from a straight railroad track on which a train is traveling at a constant speed of $60 \frac{km}{h}$. How fast is the distance between the train and the observer decreasing when the train is 5km from the observer?
32. A dog chases a squirrel at a speed of 4m per second. The squirrel dashes up a tree trunk at the rate of 2m per second. Find the rate of change of distance between squirrel and dog at an instant when the dog is 12m from the tree trunk and the squirrel is 5m up the trunk.
33. A rock is dropped into a pond, and waves begin to spread. The radius of the circle of waves grows at the rate of .5m per second. Find the rate of increase of area of the circle at the instant the radius is 8m.
34. Two parallel sides of a rectangle are being lengthened at $2 \frac{cm}{s}$, while the other two sides are being shortened such that the figure remains a rectangle with constant area of 50 cm^2 . What is the rate of change of the perimeter when the length of an increasing side is 5cm?, 10cm?
35. A square plate of metal is expanding under heat so that the side is increasing at the uniform rate of $0.1 \frac{m}{hr}$. Find the rate of increase of the area of the plate when the side is $1 \frac{1}{3} \text{ m}$.
36. In an expanding circular plate, what is the value of the radius when the area is increasing twice as fast as the radius?
37. On a baseball diamond that is 30m square, Homer Runn is dashing from second base to third at a speed of 5m per second. How rapidly is the distance from Homer to home plate decreasing at the instant he is halfway between second and third base?

Answers:

1. $\frac{4}{3\pi} \frac{m}{min}$ 2. $3.6 \frac{m}{s}$ 3. $\frac{\sqrt{3}}{2} \frac{m}{s}$ 4. $\frac{40}{9\pi} \frac{cm}{s}$ 5. $-\frac{1}{3\pi} \frac{cm}{s}$
6. $\frac{50}{81\pi} \frac{cm}{s}$ 7. $\frac{1}{2\pi} \frac{m}{min}$ 8.a) $h = 3\sqrt{t}$ b) $\frac{1}{4} \frac{m}{min}$ c) 64 min d) $\frac{1}{16} \frac{m}{min}$
9. $(10 - 0.24\pi) \frac{m^3}{min}$ 10.a) $\frac{3\sqrt{3}}{500\pi} \frac{cm}{min}$ b) $6\sqrt{3} \frac{cm^2}{min}$ 11. $(2 + \frac{45\pi}{16}) \frac{m^3}{min}$
12. $\frac{1}{6} \frac{m}{min}$ 13.a) $-\frac{1}{400\pi} \frac{m}{min}$ b) $-\frac{1}{5} \frac{m^2}{min}$ 14. $0.18\pi \frac{m^3}{min}$ 15. $0.1326 \frac{m}{min}$
16. $\frac{3}{4} \frac{m}{min}$ 17. $\frac{25}{24} \frac{m}{min}$ 18.a) $\frac{3}{4} \frac{m}{s}$ b) $\frac{\sqrt{3}}{3} \frac{m}{s}$ c) $\frac{3}{4} \frac{m}{s}$ 19. $-\frac{3}{4} \frac{m}{s}$ 20. $\frac{20}{3} \frac{km}{h}$
- 21.a) $6 \frac{m}{s}$ b) $2 \frac{m}{s}$ 22. 7 knots 23. $\frac{33\sqrt{2}}{2} \text{ knots}$ 24.a) $\text{approach } \frac{3\sqrt{2}}{2} \text{ knots}$
- 24.b) *neither* c) $\frac{3\sqrt{2}}{2} \text{ knots}$ 25. $40.093 \frac{m}{min}$ 26. 18.6 knots 27. $56.2 \frac{km}{h}$
28. $804 \frac{km}{h}$ 29. $14 \frac{m}{s}$ 30. $3.2 \frac{m}{s}$ 31. $36 \frac{km}{h}$ 32. $2.92 \frac{m}{s}$ 33. $8\pi \frac{m^2}{s}$
- 34.a) $-4 \frac{cm}{s}$ b) $2 \frac{cm}{s}$ 35. $0.27 \frac{m^2}{s}$ 36. $\frac{1}{\pi}$ 37. $2.24 \frac{m}{s}$