MCV 4U - UNIT 4 REVIEW

1. a) The function f is graphed below. Evaluate each of the following:

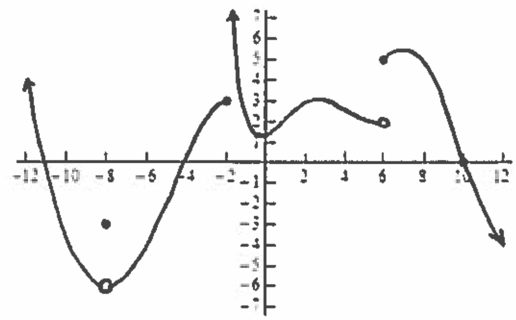
i) $\lim_{x \to -8^-} f(x) =$ ii) $\lim_{x \to -8^+} f(x) =$

iii) $f(-8) =$ iv) $\lim_{x \to -2^-} f(x) =$

v) $\lim_{x \to -2^+} f(x) =$ vi) $f(-2) =$

vii) $\lim_{x \to 6^-} f(x) =$ viii) $\lim_{x \to 6^+} f(x) =$

ix) $f(6) =$ x) $\lim_{x \to 10} f(x) =$



b) For what value(s) of x is $f(x)$ discontinuous?

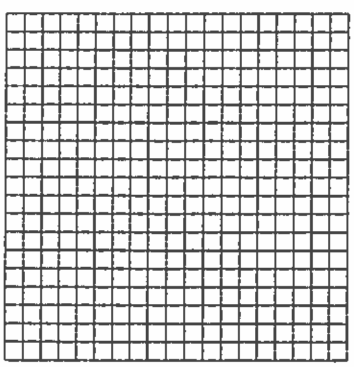
c) State the end behavior:

$\lim_{x \to \infty} f(x) =$ $\lim_{x \to -\infty} f(x) =$

2. Let $f(x) = \begin{cases} 2 - 5x^3, & x \geq 1 \\ 4 - 7x, & -3 \leq x < 1 \\ (x-2)^2, & x < -3 \end{cases}$. Is $f(x)$ continuous? Why or why not?

3. Given $g(x) = \begin{cases} x^2 - 4, & x > 2 \\ 3x - 2, & x \leq 2 \end{cases}$

a) Sketch the graph of $g(x)$.



b) Evaluate the following limits:

i) $\lim_{x \to 2^-} g(x) =$

ii) $\lim_{x \to 2^+} g(x) =$

iii) $\lim_{x \to 2} g(x) =$

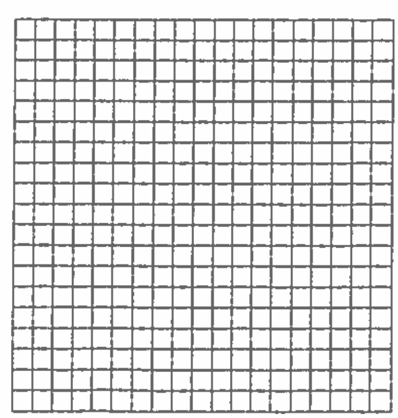
4. Explain, with the aid of a graph, what is meant by the notation: $\lim_{x \to c} f(x) = L$.

5. Sketch *any* graph for $f(x)$ satisfying **all** 6 of the following conditions:

$\lim_{x \to -3^-} f(x) = 4$ $\lim_{x \to 2^-} f(x) = 4$

$\lim_{x \to -3^+} f(x) = -3$ $\lim_{x \to 2^+} f(x) = 4$

$f(-3) = undefined$ $f(2) = 0$



6. Evaluate each limit. If the limit does not exist, state why not:

a) $\lim_{x \to 2} \frac{x^4-16}{x-2}$

b) $\lim_{x \to 3} \frac{x-3}{x^3-27}$

c) $\lim_{x \to \frac{3}{2}} \frac{|2x-3|}{2x-3}$

d) $\lim_{x \to \infty} \frac{x^3+6x^2}{2x^3+5x^2+6x}$

e) $\lim_{t \to 0} \frac{1}{t\sqrt{1+t}} - \frac{1}{t}$

f) $\lim_{x \to 0} \frac{(x+64)^{\frac{1}{3}}-4}{x}$

g) $\lim_{x \to 0} \frac{2^{x+1}+2^{-x}}{2^{\frac{1}{x}}}$

h) $\lim_{x \to 3} \frac{\sqrt{x^2+7}-\sqrt{x+13}}{x-3}$

7. After t minutes of decay, a certain radioactive element has a mass in grams of $M(t) = 100\left(\frac{1}{2}\right)^t$

a) Find the average rate of change during the first minute

b) Find the average rate of change on the interval between $t = 1$ and $t = 1.00001$.

c) Use your answer from part b to estimate the instantaneous rate of change at $t = 1$. What does this represent in the context of the problem?

8. a) What are the three conditions necessary for a function $f(x)$ to be continuous at $x = a$?

b) Each of the following functions has a discontinuity. State where it is and which type of discontinuity it is (i.e. hole, jump, or asymptote)

i) $f(x) = \frac{x^2+7x+12}{x+3}$

ii) $f(x) = -\frac{5}{3x+5}$

iii) $f(x) = \begin{cases} 3, & x \leq -2 \\ x+5, & -2 < x \leq 5 \\ \sqrt{94+x}, & x > 5 \end{cases}$

9. a) State the expression used to calculate the slope of the tangent of a function $f(x)$ at a point $x = a$.

b) Use the expression from part a) to find the slope of the tangent at $x = -2$ for each function.

i) $f(x) = x^3$

ii) $f(x) = \frac{3}{\sqrt{7-x}}$

10. Find the equation of the tangent line from #9 b) i) at $x = -2$.

11. Find constants a and b such that the piecewise function

$$f(x) = \begin{cases} -x, & -3 \leq x \leq -2 \\ ax^2 + b, & -2 < x < 1 \\ 3b + 4a, & 1 \leq x \end{cases}$$

is continuous for $x \in (-3, \infty)$.