Any questions from the homework pg 132?

Lines in $\mathbb{R}^3$

Vectors Parametric & Symmetric Eq'ns extend to $\mathbb{R}^3$
in $\mathbb{R}^2$ ⟶ in $\mathbb{R}^3$

Let $A(x_1, y_1)$
$\vec{m} = [x_{\vec{m}}, y_{\vec{m}}]$

V: $[x, y] = [x_1, y_1] + t[x_{\vec{m}}, y_{\vec{m}}]$

P: $x = x_1 + t x_{\vec{m}}$
$y = y_1 + t y_{\vec{m}}$

S: $\frac{x - x_1}{x_{\vec{m}}} = \frac{y - y_1}{y_{\vec{m}}}$

Let $A(x_1, y_1, z_1)$
$\vec{m} = [x_{\vec{m}}, y_{\vec{m}}, z_{\vec{m}}]$

V: $[x, y, z] = [x_1, y_1, z_1] + t[x_{\vec{m}}, y_{\vec{m}}, z_{\vec{m}}]$

P: $x = x_1 + t x_{\vec{m}}$
$y = y_1 + t y_{\vec{m}}$
$z = z_1 + t z_{\vec{m}}$

S: $\frac{x - x_1}{x_{\vec{m}}} = \frac{y - y_1}{y_{\vec{m}}} = \frac{z - z_1}{z_{\vec{m}}}$

* if a fixed point has a zero for a coordinate
e.g. $(0, y, z)$ is on the yz plane
$(x, 0, z)$ is on the xz plane
$(x, y, 0)$ is on the xy plane

* if $\vec{m}$ has a zero coordinate eg $[x_{\vec{m}}, y_{\vec{m}}, 0]$
then the line is // to the xy plane

* if the direction vector has a zero component
the symmetric eq'ns cannot be written.

ex 1. Given $P(2, 4, 0)$ & $Q(5, 0, 7)$
Find for the line passing through PQ
i) vector eq'n
ii) parametric eq'ns
iii) symmetric eq'ns

Find $\vec{m} = \vec{PQ}$
$= [3, -4, 7]$
using P as fixed point.

i) $[x, y, z] = [2, 4, 0] + t[3, -4, 7]$

ii) $x = 2 + 3t$
$y = 4 - 4t$
$z = 7t$

iii) $\frac{x - 2}{3} = \frac{y - 4}{-4} = \frac{z}{7}$

ex2
Given the following symmetric eq^{ns}

$$\frac{x-3}{2} = \frac{y+4}{-1} = \frac{z-1}{3}$$

a) write the parametric eq^{ns}
b) Find 3 different points on the line.

a) $x = 3+2t$
$y = -4-t$
$z = 1+3t$

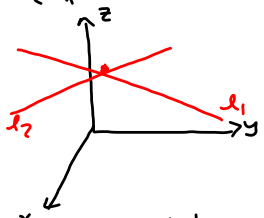
b) choose different t values

$t=1$ $(5,-5,4)$
$t=2$ $(7,-6,7)$
$t=0$ $(3,-4,1)$

What about 2 lines in $\mathbb{R}^3$ intersecting?

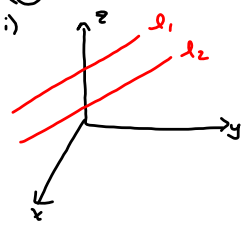
There are 3 possible outcomes

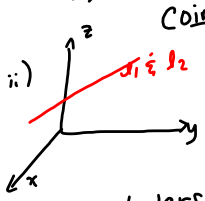
① Intersect at one point (& are on the same plane)



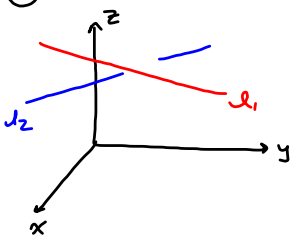
② Parallel Lines i) No intersection if distinct

ii) Infinite # intersection if coincident





③ Neither Parallel nor Intersect SKEW



ex3 Do those eq'ns represent the same line?

$l_1: \frac{x-5}{2} = \frac{y+4}{3} = \frac{z+1}{-5}$ $l_2: \frac{x+1}{-4} = \frac{y-1}{-6} = \frac{z+3}{10}$

Step 1 // or not?

Compare the direction vectors

$\vec{m}_1 = [2,3,-5]$ $\vec{m}_2 = [-4,-6,10]$

$\div -2$

$= [2,3,-5]$

Since $\vec{m}_2 = -2\vec{m}_1$

$l_2 // l_1$

Step 2 Distinct or coincident?

Check the fixed point from l_1 in the symmetric eq'ns of l_2 if we get the same ratio then the lines are the same.

fixed pt l_1 $(5,-4,-1)$

symmetric l_2 $\frac{x+1}{-4} = \frac{y-1}{-6} = \frac{z+3}{10}$

$\frac{5+1}{-4} = -\frac{3}{2}$

$\frac{-4-1}{-6} = \frac{5}{6}$

$\frac{-1+3}{10} = \frac{1}{5}$

} Not the same

$\therefore l_1$ & l_2 are parallel and distinct.

ex4 Do these lines intersect? If so, @ what point?

$l_1: \frac{x-3}{1} = \frac{y+7}{-2} = \frac{z-5}{4}$ $\quad \vec{m}_1 = [1,-2,4]$

$l_2: \frac{x+7}{3} = \frac{y+8}{1} = \frac{z-4}{-1}$ $\quad \vec{m}_2 = [3,1,-1]$

$\vec{m}_1 \neq k\vec{m}_2$ $\therefore$ l_1 & l_2 not parallel

switch to parametric

l_1: $x = 3+t$, $y = -7-2t$, $z = 5+4t$

l_2: $x = -7+3s$, $y = -8+s$, $z = 4-s$

if they intersect

$3+t = -7+3s$ ①

$-7-2t = -8+s$ ②

$5+4t = 4-s \longrightarrow s = -1-4t$ ③

sub into ②

$-7-2t = -8+(-1-4t)$

$-7-2t = -9-4t$

$t = -1$

$\therefore s = 3$

Check in eqn ①

LS	RS
$3+t$	$-7+3s$
$3-1$	$-7+9$
2	2

✓ yes

* if this did NOT work Lines are SKew

$\therefore$ find (x,y,z)

$x = 2$

$y = -5$

$z = 1$

So the POI is $(2,-5,1)$

Homework: pg 141 #4,5,9-15, 22-24

Quiz Thursday
on lines in $\mathbb{R}^2$ & $\mathbb{R}^3$