LONDON CENTRAL SECONDARY SCHOOL
MATHEMATICS DEPARTMENT

MCA OA1

CALCULUS EXAMINATION

JUNE 1991
TIME: 2.5 hours
Please answer all questions on foolscap.
Graphing calculators are not permitted on this examination.

1. Use first principles to determine the derivative of the following function with respect to x ...

$$y = 2x - \frac{1}{x}$$ /6

2. Evaluate each of the following limits.
If a limit does not exist, then so indicate.

/10

a) $\lim\limits_{x \to -1} \dfrac{3x^2 - x - 4}{x^3 - x}$

b) $\lim\limits_{x \to \infty} \dfrac{4x^5 - 3x^2}{6x + 2 - 8x^5}$

c) $\lim\limits_{x \to 3} \dfrac{\frac{3}{x} - 1}{x^2 - 9}$

d) $\lim\limits_{x \to \infty} \left[3(10^{\frac{4}{x}}) + 5(-2)^{-x}\right]$

~~e) $\lim\limits_{\theta \to 0} \dfrac{2\sin 3\theta}{5\theta}$~~

3. Determine, without simplifying, the **derivative of y with respect to x** for each of the following:

/12

a) $y = \dfrac{3}{\sqrt[4]{x}} + \dfrac{2}{x^{-3}}$

b) $y = \sqrt[3]{\dfrac{2x}{x^2 + 4}}$

c) $y = (2 + 3x^3)^5 (4 - 5x^2)^{-3}$

d) $y = xe^{(2-x)} + \ln(\cos x)$

e) $y = 3\sin^4(e^{\sqrt{x}})$

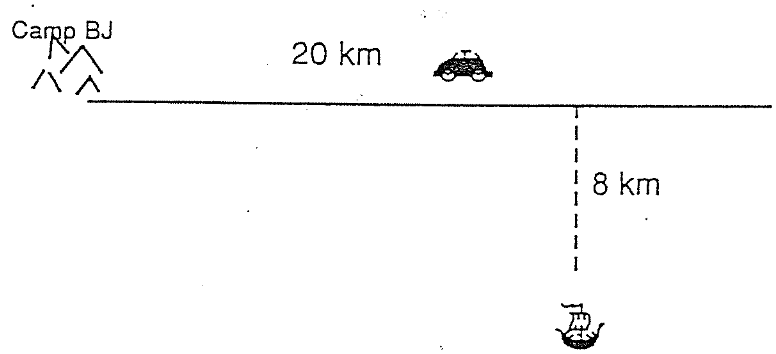
4. Determine the equation of the tangent to the following curve at the point (1,1). Express your answer in the form $ax + by + c = 0$

$$x^2 + 3x^2y^2 + 2y^2 = 6$$ /5

5. A conical container has a depth of 12 m. It's top is a circle of radius 4 m. Initially the container is empty. Then water is poured into the container at a constant rate of 10 m^3/ min. Meanwhile, it is noted that 2 m^3 of water is leaking from the container every minute. Find the rate (to 1 decimal place accuracy) at which the level of the water is rising in the container at the instant that the volume of water in the container is 8π cubic meters.

$$V = \frac{1}{3}\pi r^2 h$$ /6

6. A messenger is to go ashore from a ship located 8 km from a straight shore and deliver a message to a camp located 20 km up the beach from the point on the shore nearest the ship. From the ship, he can travel by boat at 15 km/h. He will be met on shore by a jeep which can travel at 25 km/h over the beach. Where (to the nearest km) should he land on shore to complete the trip in the shortest possible time?



/6

7. Sketch the curve given by $y = x^3 - 3x^2$
Show your calculations for maximum points, minimum points, points of inflection and intercepts and asymptotes.
Your sketch must be labelled and indicate the features calculated above. /8

8. Consider the curve ... $y = 2\cos X + \sin 2X$ for $0 \le X \le 2\pi$
Calculate each of the following:
a) one maximum point in the given interval
b) one minimum point in the given interval
c) all x intercept(s) in the given interval
(Note ... you are not required to graph the curve) /10

9. Simplify ...
$\sin(-B) + \sin(270^\circ - B) + \sin(180^\circ - B) + \sin(90^\circ + B)$
where B is any real number. /5

Find $\frac{dy}{dx}$

a) $y = \frac{3}{\sqrt[4]{x}} + \frac{2}{x^{-3}}$ /12

b) $y = \sqrt[3]{\frac{2x}{x^2+4}}$

c) $y = (2+3x^3)^5(4-5x^2)^{-3}$

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